Games and Economic Behavior 115 (2019) 459–469

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

# Measuring time and risk preferences in an integrated framework ☆

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ARTICLE INFO

*Article history:*
Received 27 July 2017
Available online 19 March 2019

*JEL classification:*
D81
D90

*Keywords:*
Time preferences
Risk preferences
Rank-dependent utility
Present bias

ABSTRACT

We investigate time discounting under risk. To this end, we modify a popular multiple price list (*MPL*) design to elicit time discounting. Structural estimations of model parameters yield several new insights. For one, we find present bias to persist under risk, contrary to some previous evidence from the psychology literature. We further confirm the robustness of inverse-S shaped probability weighting. This is important inasmuch as random choice predicts the opposite shape in our setup. We also show that correcting for probability weighting under risk impacts the assessment of discount rates. Those are systematically underestimated under the commonly used, more restrictive, expected utility.



## 1. Introduction

Risk and time are fundamentally intertwined—the future is inherently risky. Yet time preferences have mostly been studied while abstracting from risk under presumed certainty (see Frederick et al., 2002 for a review). Indeed, it has been suggested that deviations from the standard model of inter-temporal decision making, discounted utility with an exponentially decreasing discount function (*DU*; Samuelson, 1937), may be largely or entirely due to elicitation methods positing certainty of future outcomes (Keren and Roelofsma, 1995; Weber and Chapman, 2005; Halevy, 2008; Gerber and Rohde, 2010; Epper et al., 2011). According to this suggestion, (quasi-) hyperbolic preferences (Phelps and Pollak, 1968; Laibson, 1997; Rohde, 2010; Pan et al., 2015) are due to the absence of risk in the present, whereas risk is inherent in any future outcomes. A dislike of risk would then result in a preference for immediate outcomes over future ones, regardless of a respondent's true underlying discount rate.

☆ The experiment reported in this paper was financed by the WZB Berlin Social Science Center. Ferdinand Vieider gratefully acknowledges financial support received from the German Science Foundation (DFG) under project VI 692/1-1.

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https://doi.org/10.1016/j.geb.2019.03.001

This paper investigates the properties of time discounting under risk. We do so using a novel method consisting of a simple variation on the specific type of multiple price lists (MPLs) popularized in economics by Holt and Laury (2002).[1] We start by using standard MPLs to elicit risk preferences. That is, we compare two binary lotteries while changing the probabilities attached to the different outcomes in a choice list. By eliciting the switching probability between a (relatively) risky and a (relatively) safe lottery, we identify respondents' preferences over risk. We can then identify time preferences simply by differing the payouts of one of the lotteries into the future (the resolution of uncertainty is always immediate). By always deferring the outcomes of the *safe* lottery we create a psychological tradeoff between preference for the present and risk aversion, since the price to pay for increased safety is a delay in the payout of the outcome. By administering appropriate delays of both lotteries to different future dates, we can further measure quasi-hyperbolic and hyperbolic discounting. In addition, we show how to use MPLs to elicit probability weighting jointly with utility curvature. Previous studies using this particular choice list design were generally not set up to do this (we will return to this point in the discussion). This serves as a stability check of the typical inverse-S shape of probability weighting (see van de Kuilen and Wakker, 2011, for an overview). While different methods have been used to measure probability weighting (see e.g. Abdellaoui, 2000, and Bleichrodt and Pinto, 2000, for non-parametric measurements), many studies have employed certainty equivalents (*CEs*) to parametrically identify utility and probability weighting functions (Tversky and Kahneman, 1992; Bruhin et al., 2010; Abdellaoui et al., 2011; l'Haridon and Vieider, 2018). In these tasks, lotteries with a given probability of winning a prize are compared to a series of sure amounts of money in a choice list. In the choice list, the CE is derived from the point at which people switch from the sure amount to the lottery. While being eminently tractable, CEs can be biased by systematic noise. While this problem has been known for many years, some recent studies have highlighted the fact that some people may switch systematically in the middle of a list, or at random (Ert and Erev, 2013; Andersson et al., 2016; Vieider, 2018). Using CEs, this kind of switching pattern could result in inverse-S shaped probability weighting even if respondents were in fact expected utility maximizers. However, with the MPLs used here, this switching pattern would result in S-shaped probability weighting, thus providing a stability test for inverse-S shaped weighting.

Estimating probability weighting in addition to utility curvature further allows us to examine the effect of the risky-choice model adopted on the estimated discount function. We start from the estimation of the standard model of inter-temporal decision making in the presence of risk, discounted expected utility (*DEU*). We then relax its assumptions by allowing for non-constant discounting and non-linear probability weighting, both of which substantially improve the fit of the model to the data. Accounting for non-linear probability weighting is also important for another reason. In the presence of pessimism in the probability weighting function, utility obtained while assuming EU will be overly concave (Wakker, 1994). Correcting for probability weighting resolves this problem, and thus results in reduced concavity of utility (Bleichrodt et al., 2007; Schmidt and Zank, 2008). The risky-choice model assumed will also influence estimations of time discounting. This is, indeed, a direct consequence of imposing the utility parameter estimated under risk for modeling inter-temporal tradeoffs.

We find that probability weighting is indeed inverse-S shaped, thus confirming the stylized fact of probabilistic insensitivity—the finding that a given change in probability receives considerably less weight when it takes place in the interior of the probability interval than when it occurs towards the probability end-points of 0 and 1. This proves the robustness of this finding of inverse-S probability weighting to the potential confound of random switching. We also reject constant discounting in favor of hyperbolic discounting. Estimating a DEU model with constant discounting and linear probabilities, we estimate a low yearly discount rate of around 6%. Once we allow for non-linear probability weighting, however, the estimated discount rate more than doubles to 14%. This dramatic change is due to the fact that utility estimated in conjunction with probability weighting exhibits considerably less curvature than utility estimated under the expected utility assumption. This shows that correcting discount rates for utility measures obtained from risky choices under the assumption of expected utility maximization may lead to the systematic underestimation of discounting. We will further discuss these insights after presenting the results.

## 2. Experimental design and model estimation

*Subjects* We recruited 100 subjects at the laboratory of the Technical University in Berlin, Germany. The students were from a variety of study majors, 41% were female, and the average age was approximately 22 years. The experiment was computerized and run in 20 small group sessions of five participants each. The average duration of the experiment was 45 minutes.

*General choice setup* The subjects were presented with two dated lotteries for each MPL, as shown in Fig. 1. The lotteries were such that $x_{r,t} > x_{s,t+\tau} > y_{s,t+\tau} > y_{r,t}$. Consequently, the lottery on the left-hand side has a higher spread in outcomes than the lottery on the right-hand side, making it more risky according to the definition of riskiness by Rothschild and Stiglitz (1971), so that we subscript its outcomes by $r$. The lottery on the right-hand side will be referred to as the *safe* lottery, with its outcomes subscripted by $s$ (the terms safe and risky were not used to refer to the lotteries during the

[1] Many versions of choice lists have been used for many years. The one used here has two non-degenerate lotteries with two probabilities changing as one moves down the list. Farquhar (1984) surveys different choice lists designs.

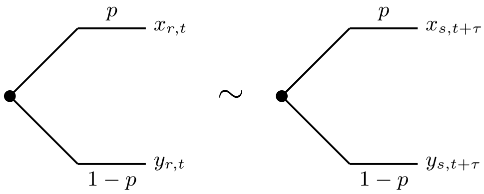



**Fig. 1.** General choice setup.

experiment). The subscripts $t$ and $t+\tau$ indicate when the outcomes of the lottery will be paid. To elicit risk preferences, we set $t=\tau=0$, so that all payouts take place in the present. Delays in payouts were introduced by setting $\tau>0$. Up-front delays were also introduced by using $t>0$ so as to test for hyperbolic behavior. The elicitation task consisted in finding the probability with which subject would switch their preference from the safe lottery to the risky one. The procedures used are described below.

*Decision model* We then describe our modeling assumptions. We start with a discounted expected utility (*DEU*) model, in which a subject chooses the risky lottery if:

$$pD(t)u(x_r)+(1-p)D(t)u(y_r)\geq pD(t+\tau)u(x_s)+(1-p)D(t+\tau)u(y_s), \tag{1}$$

where $u$ indicates utility, and $D(t)=e^{-rt}$ the exponential discount function with constant discount rate $r$. We also consider two extensions to this model. In one, the linear treatment of probabilities in equation 1 is replaced by non-linear probability weighting, thus substituting $w(p)$ for $p$. The other allows for more general functional forms for discounting, $D(t)$, which can capture non-constant discount rates.

*Functional forms* For utility, we employ a simple power function, $u(x)=x^{\rho}$, namely the constant relative risk aversion (CRRA) specification. This commonly used function provides a good fit to our data. For probability weighting, we use the 2-parameter function proposed by Prelec (1998), $w(p)=\exp(-\eta(-log(p))^{\gamma})$. This function fits the data better than 1-parameter functions such as the one proposed by Tversky and Kahneman (1992) ($z=16.4$, $p<0.001$; Vuong Quang, 1989, test), or the 1-parameter version of the same function obtained by imposing $\eta=1$ ($\chi^2(1)=13.23$, $p<0.001$; likelihood ratio test). Other 2-parameter functions, such as the one proposed by Goldstein and Einhorn (1987), provide a similar fit to the data and yield similar results. Each of the two parameters of the weighting function has a specific interpretation, with $\gamma$ capturing mostly the curvature of the weighting function. More specifically, the values $\gamma<1$ indicate inverse-S shaped probability weighting, $\gamma=1$ perfect probabilistic sensitivity, and $\gamma>1$ S-shaped weighting. The parameter $\eta$ indicates (mainly) the elevation of the weighting function, with $\eta>1$ capturing the typical case of probabilistic pessimism. Finally, the so-called $\beta-\delta$ function is used for capturing quasi-hyperbolic discounting, resulting in the following functional form for discounting:

$$D(t)=\begin{cases}1, & \text{if } t=0,\\ \beta e^{-rt}, & \text{otherwise.}\end{cases}$$

For $\beta=1$, the function above reduces to the exponential discount function of DEU. Values of $\beta<1$ capture systematically lower valuations of future outcomes in relation to present outcomes. In addition, we also fit a fully hyperbolic discounting function proposed by Loewenstein and Prelec (1992) to the data. The function takes the form $D(t)=(1+\zeta t)^{\frac{-r}{\zeta}}$, where the $\zeta$ parameter captures the degree of deviation from exponential discounting. The limit of this specification as $\zeta$ tends to 0 is the exponential discounting function. Last, we use the constant-sensitivity function proposed by Ebert and Prelec (2007). It takes the form $D(t)=exp(-(at)^b)$, where $a$ measures impatience, and $b$ measures time-sensitivity. For $b=1$ the function is reduced to exponential (constant) discounting, while for $b<1$ the function takes a hyperbolic form. Interestingly, the values of $b>1$ can also accommodate patterns of increasing impatience.

*Stochastic specification and econometrics* Potential noise in the data is taken into account by incorporating an error term, $\epsilon_i$. Writing the valuation of the risky lottery as $U_r$ and the valuation of the safe lottery as $U_s$, a subject will choose the *risky* lottery if $U_r\geq U_s+\epsilon_i$. We assume $\epsilon_i$ to be normally distributed (Hey and Orme, 1994), $\epsilon_i\sim N(0,\sigma_i^2)$. We further allow the error term to depend on characteristics of the specific MPL, indexed by $i$. In particular, we allow the error term to depend linearly on the outcome range in the risky prospect, $x_r-y_r$, which provides a good fit to our data (see also Bruhin et al., 2010). The choice probability can then be written as

$$P(\text{choose risky})=P(\epsilon_i<U_r-U_s)=\Phi\left(\frac{U_r-U_s}{\sigma_i}\right), \tag{2}$$

**Table 1**
Prospect pairs to identify risk preferences.

| MPL nr. | outcomes in € | EV prob. |
|---|---|---|
| 1 | (250, 10) vs. (200, 50) | 0.44 |
| 2 | (300, 20) vs. (200, 100) | 0.44 |
| 3 | (500, 0) vs. (250, 200) | 0.44 |
| 4 | (500, 20) vs. (400, 100) | 0.44 |
| 5 | (500, 220) vs. (400, 300) | 0.44 |
| 6 | (500, 10) vs. (150 , 50) | 0.10 |
| 7 | (500, 220) vs. (300, 250) | 0.13 |
| 8 | (450, 150) vs. (250, 200) | 0.20 |
| 9 | (500, 10) vs. (450, 50) | 0.44 |
| 10 | (350, 50) vs. (250, 200) | 0.60 |
| 11 | (500, 0) vs. (350, 300) | 0.67 |
| 12 | (500, 0) vs. (400, 350) | 0.78 |

where $P$(choose risky) indicates the probability of choosing the risky lottery, and $\Phi$, the cumulative normal distribution function. The model can now be estimated by maximum likelihood. To obtain the overall log-likelihood function, we take the natural logarithm of the cumulative distribution function in equation 2 and aggregate it over prospects and decision makers as follows:

$$LL(\boldsymbol{\theta}) = \sum_{n=1}^{N}\sum_{i=1}^{43} \ln\left(\mathbb{1}_{\mathrm{r}}\Phi\left(\frac{U_r - U_s}{\sigma_i}\right) + (1-\mathbb{1}_{\mathrm{r}})[1-\Phi\left(\frac{U_r - U_s}{\sigma_i}\right)]\right) \tag{3}$$

where $\mathbb{1}_{\mathrm{r}}$ is an indicator variable that is equal to 1 if the risky prospect is chosen, and to 0 if the safe prospect is chosen, and $\boldsymbol{\theta}$ is the parameter vector to be estimated by maximizing the log-likelihood function. The likelihood model is estimated using the Broyden-Fletcher-Goldfarb-Shanno optimization algorithm and errors are clustered at the subject level. Parameters are constrained to be greater than 0 for the individual-level estimations.

*Identification of risk preferences* We identify risk preferences from choices involving lotteries with payouts in the present ($t = \tau = 0$). Table 1 shows a list of the MPLs used for the elicitation. MPLs 1 to 5 keep the expected value switching probability (i.e., the probability at which an expected value maximizer would switch from the safe lottery to the risky one) fixed at 0.44—the switching probability originally used by Holt and Laury (2002). These MPLs were constructed to differ in terms of outcomes, allowing us to scan the outcome range and thus identify utility curvature. On the other hand, we constructed prospect pairs 6 to 12 so as to scan the interval of expected value ($EV$) switching probabilities.[2] While other studies have tried to estimate probability weighting using similar MPLs, the range of EV switching probabilities was too narrow to properly separate utility curvature from probability weighting. For instance, Andersen et al. (2014) used four MPLs with a range of 0.30 to 0.45, and found an S-shaped probability weighting, as did other estimations presented by the same authors (see e.g. Andersen et al., 2017). Their design does not have the power to clearly identify probability weighting. Indeed, it is known that probability weighting functions tend to be relatively flat and close to linearity for the EV switching probability range they used. This problem may be further confounded by noise in the data. By systematically introducing variation in EV switching probabilities, we solve this issue and augment the power to properly identify probability weighting and utility curvature.

Below we present a quick review of the matter of random switching. Assume that some subjects switch at random points in a list (the tendency to switch towards the middle of a list results in the same prediction). On average, these subjects will exhibit a switching probability of 0.5. Now take MPL 6. Since a risk neutral respondent would switch to the risky lottery at $p = 0.1$, a risk seeker would switch to that lottery at an even lower probability. However, given that the choice list ranges over the whole probability interval, random switching behavior would result in an estimate of risk *averse* behavior. Conversely, for MPL 12, a risk averse subject would switch to the risky lottery only once the probability is above 0.78. For this MPL, random choices would be counted towards risk *seeking*. We conclude then that, in the current setup, systematic noise in the form of random switching would result in an S-shaped probability weighting function. The exact opposite occurs for CEs, where random choice is potentially confounded with inverse-S probability weighting, thus highlighting the importance of systematic noise in the identification of probability weighting.

*Identification of time preferences* Time preferences are identified by delaying the payouts of the lotteries into the future (the uncertainty is always resolved immediately after the experiment). Table 2 provides an overview of the choice tasks used to identify time preferences. The EV switching probability is now fixed at a constant of 0.44. Each of the different MPLs

[2] This is done by systematically adjusting the outcome spread of the two prospects. Let $k = \frac{x_r - x_s}{y_s - y_r}$. We can then compute the expected value switching probability of the MPL with $p(EV) = \frac{1}{1+k}$. It is now straightforward to manipulate $k$ to obtain EV-switching probabilities $p(EV)$ that scan the probability interval.

**Table 2**
Prospect pairs to identify time preferences.

| MPL nr. | outcomes in € | EV prob. |
|---|---|---|
| 1 | $(250_t, 10_t)$ vs. $(200_t + \tau, 50_t + \tau)$ | 0.44 |
| 2 | $(300_t, 20_t)$ vs. $(200_t + \tau, 100_t + \tau)$ | 0.44 |
| 3 | $(500_t, 0_t)$ vs. $(250_t + \tau, 200_t + \tau)$ | 0.44 |
| 4 | $(500_t, 20_t)$ vs. $(400_t + \tau, 100_t + \tau)$ | 0.44 |
| 5 | $(500_t, 220_t)$ vs. $(400_t + \tau, 300_t + \tau)$ | 0.44 |

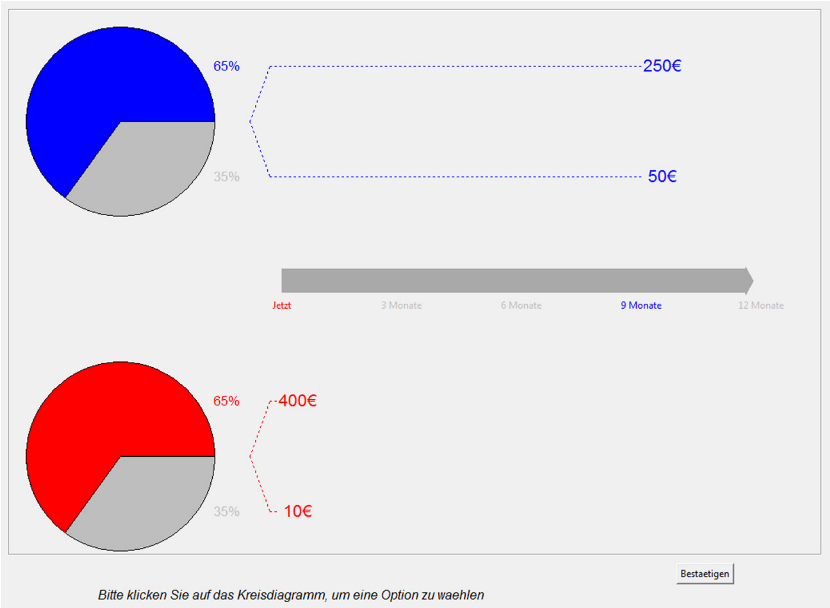



**Fig. 2.** Screenshot from the experiment's time preference section.

is repeated for each of the time delays $(t, t + \tau) = \{(0, 3); (0, 6); (0, 9); (0, 12); (6, 12); (9, 12)\}$ months. By comparing the lottery choice resulting from $t = 0, \tau > 0$ to the equivalent choice for $t = 0, \tau = 0$, we obtain an estimate of discounting. By comparing choices in MPLs with constant delays, $(0, \tau)$ and $(t, t + \tau)$, we can then determine whether the discount rate is constant, or whether it follows a hyperbolic pattern.

*Choice procedures* The experiment consisted of 42 different choice lists. Three of these lists were randomly selected for each subject and repeated during the experiment, so that subjects completed a total of 45 choice lists. Some of the lists were presented several times to determine the consistency of behavior, and to help identify the error term in the structural estimations.[3] The order of questions was randomized at the subject level. The pay-off amounts remained fixed in each list but the probabilities varied in 5% increments across each row. In order to focus the subjects' attention, the choices were presented one by one. A screenshot of a choice problem is shown in Fig. 2. The display shows a choice between a risky lottery, offering either €400 with a probability of 0.65 or €10, both with payoffs in the present, against a safe lottery offering the same probability of €250 or €50 to be paid in 9 months. The probability of winning was adjusted according to the choice using a bisection mechanism. However, subjects were clearly informed that the mechanism served only as a decision aid to speed up the process of filling the choice list. Once all the choices for a given list had been made and the list was fully completed, subjects were shown the complete choice list and explicitly encouraged to amend their choice in case they were not happy with it. Importantly, it was made clear to them that the full list would be used for the final extraction of the payout-relevant choice, with all choices equally likely to be selected.

*Incentives and randomization* Subjects were paid a fixed amount of €15 for their participation. In addition, we used a random incentive mechanism whereby each subject had a 1 in 10 chance of receiving payment for their choices. This allowed us to use high monetary stakes ranging up to €500. Such high amounts are important for the estimation of utility functions, since small amounts would be expected to generate little curvature. They are also important for time discounting, since low stakes have been found to result in inflated discount rates (the *magnitude effect*; Loewenstein and Thaler, 1989).

[3] The test-retest reliability of our measures, defined as the correlation between responses in identical tasks, was between 0.75 and 0.85, and thus falls into the typical range observed in this type of experiment.

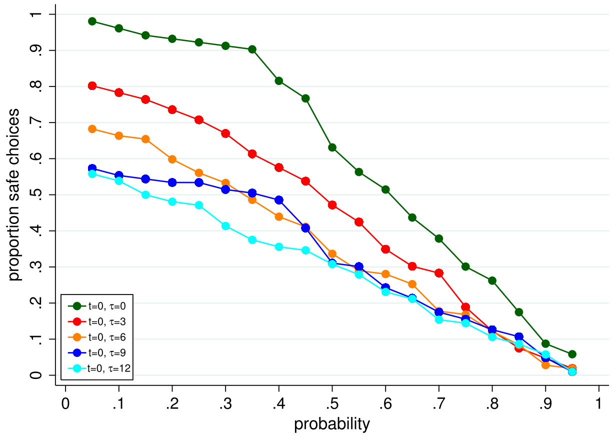



**Fig. 3.** Choices for MPL (500, 220) vs (400, 300) with different delays from $t = 0$.

Subjects were informed that if they were selected to play the tasks for real money, one of the choice lists would be selected at random. Within that choice list, one probability would then be selected, and the lottery of their choice would be played out for that probability.

*Delayed payouts* The participation fee of €15 was paid as soon as the experiment was completed. All other payouts were made by bank transfer initiated at time $t$ or $t + \tau$. This meant a fixed upfront delay of 3 days between the date indexed by $t$ and the day the subject would actually receive the money, for the sake of consistency the same rule was also applied to later dates.[4] This served to address concerns that any present bias observed may have been driven by the immediacy of the current payoff, or by differences in transaction costs between immediate and delayed payoffs (Coller and Williams, 1999). All payments were guaranteed by the WZB Berlin Social Science Center, which was familiar to participants as it is one of the institutions running the lab. Subjects were given a certificate signed by the experimenter indicating the amount won and the day on which the transfer would take place. The certificate also specified the address, email address, and telephone number of the person at the WZB responsible for the payouts. Subjects were explicitly encouraged to get in touch in case their bank details changed, or if they had any questions about the payout procedure.

## 3. Results

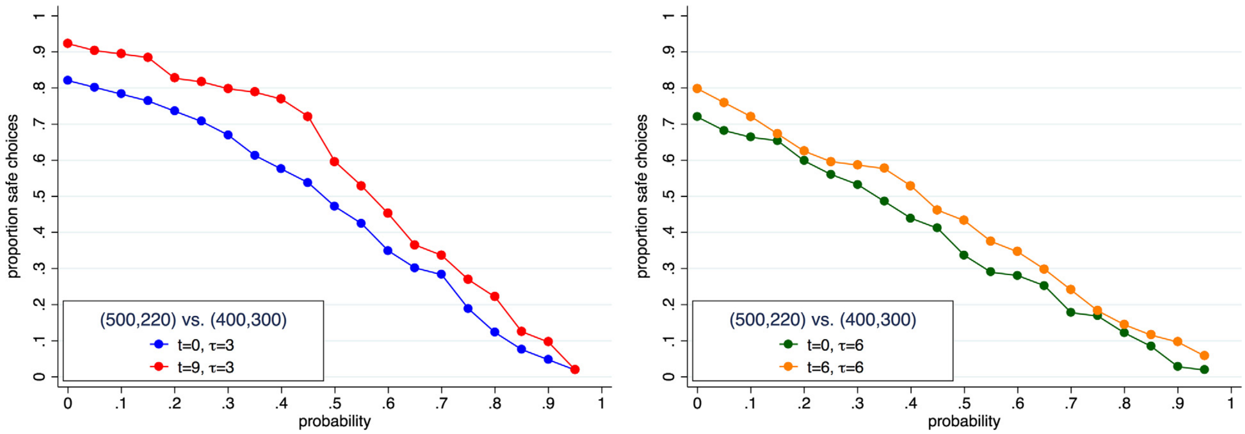
### 3.1. Non-parametric results

Our analysis begins with some non-parametric results that give an idea of our main findings. We start by discussing the effect of delaying the safe lotteries into the future. Fig. 3 focuses on one specific series of MPL, (500, 220) vs. (400, 300), and presents their choice distributions at the 5 different time delays from $t = 0$ (results for other MPLs with $t > 0$ are similar). The proportion of safe choices at different probabilities is highest in the present. As choices are delayed into the future, subjects choose the risky, sooner option more frequently, as would be expected. For the longest delay of $\tau = 12$ months, 60% of subjects prefer the risky, sooner lottery even when there is a 0% probability of obtaining the high outcome. This indicates a preference for €220 now over €300 in 12 months' time, thus implying a yearly discount rate of 36% or more under a linear utility assumption (which we will relax in due time).

Next, we take a look at whether discount rates are constant or whether there is an indication of (quasi-) hyperbolic behavior in our data. Fig. 4 shows comparisons between choices in pairs of MPLs that can be used to identify this kind of behavior. Fig. 4(a) shows choices for the MPLs with a 3 month delay from the present versus a 3 month delay from 9 months, while Fig. 4(b) shows choices for the MPL with a 6 months delay from the present versus the MPL with a 6 months delay from 6 months. Under constant discount rates, we would expect these two pairs to show identical choice patterns. Present bias, on the other hand, would make the risky lottery more attractive when there is no upfront delay (i.e., when $t = 0$). This is indeed what we observe, providing a first indication of present bias in our data.

We now move on to describing behavior under risk. We start by examining choice behavior in the MPLs by scanning the probability interval. Fig. 5 plots choices for lottery pairs 6 to 12 from Table 1. We would expect the proportion of safe choices to drop off more quickly for MPLs with a lower expected value switching point. This is indeed almost always the case. We can also use the choices to get an idea of whether risk preferences might change with the level of the EV switching point. For example, for the prospect pair (500, 220) vs. (300, 250), with an EV switching probability of $p = 0.13$, the proportion of safe choices drops quickly and, at the EV probability, about 50% of participants have abandoned the safe option. This is an

[4] We did not introduce payout delay into our model, so that the $\beta$ parameter in the quasi-hyperbolic model would remain identified.




(a) Choice patterns for 3 months delays (b) Choice patterns for 6 months delays

**Fig. 4.** Non-parametric test of (quasi-) hyperbolic behavior.

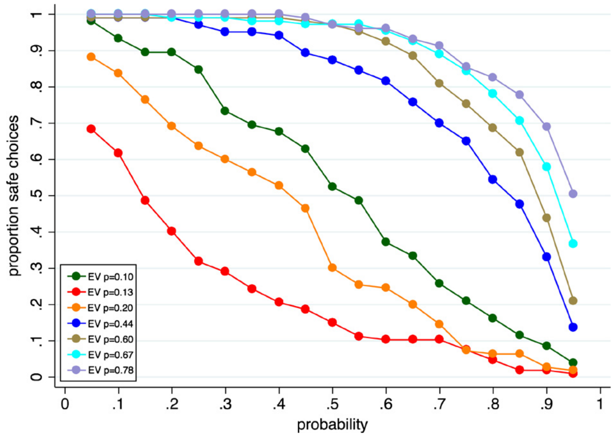


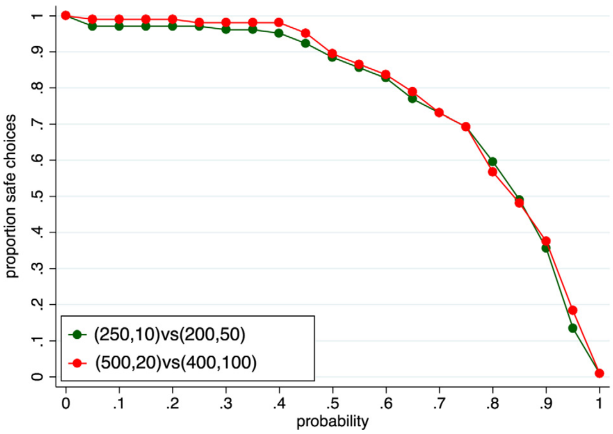

**Fig. 5.** Choice lists in the present spanning a range of expected value switching points.

indication of risk neutral behavior. At the other extreme of the probability interval, for the prospect pair (500, 0) vs. (400, 350) with an EV switching probability of $p = 0.78$, close to 90% of subjects are still choosing the safe prospect at the EV probability—an indication of considerable risk aversion for large probabilities. At the same time, however, we also observe considerable heterogeneity in choice behavior between MPLs with relatively similar EV switching probabilities. This points to the importance of utility curvature in addition to probability weighting.

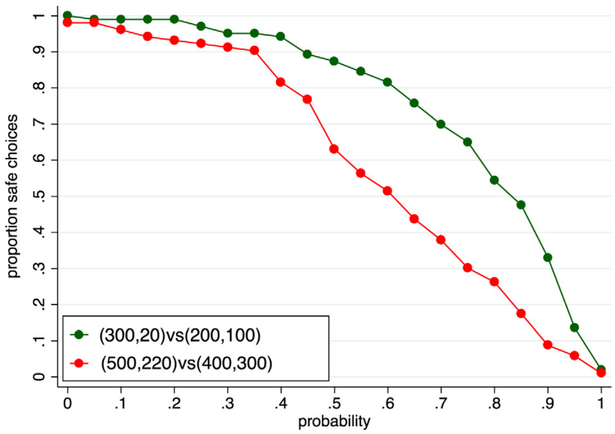
Fig. 6 shows two plots, which together constitute a test of whether utility follows constant relative risk aversion (*CRRA*, i.e. a power function) or constant absolute risk aversion (*CARA*; i.e. an exponential function). If utility exhibits CRRA, the choice patterns for the two MPLs shown in Fig. 6(a), (500, 20) vs. (400, 100) and (250, 10) vs. (200, 50), should be identical. This is because the first MPL can be obtained from the second by doubling all outcomes, so that the relative risk remains constant across MPLs. A similar test is shown for CARA in Fig. 6(b). Here one of the MPLs, (500, 220) vs. (400, 300), is obtained from the other, (300, 20) vs. (200, 100), by adding a fixed amount of €200 to each outcome. Choice behavior should be the same in the two MPLs if subjects exhibit CARA utility because of the exponential form of the utility function. The distributions of choices in the CRRA comparison coincide almost perfectly ($z = 0.293$, $p = 0.770$, Mann-Whitney test on switching probabilities). In the CARA comparison, on the other hand, the proportion of safe choices is always lower and drops off more sharply for the second MPL ($z = 4.78$, $p < 0.001$).

### 3.2. *Parametric estimations*

Table 3 presents the results of our structural estimations. Column 1 presents the DEU model, assuming linear probabilities and constant discounting. We find a considerable degree of utility curvature, while the yearly discount rate is estimated to be quite low: 5.9%. The second column reports parameters for what we call the discounted rank-dependent utility model (*DRDU*). This model combines constant discounting with a model under risk allowing for both utility curvature and the non-linear weighting of probabilities. This results from applying a probability weighting function $w$ to the probability $p$ in equation 1. The functional fit is improved considerably compared to the DEU model ($\chi^2(2) = 4094.83$, $p < 0.001$; likelihood



(a) CRRA test



(b) CARA test

**Fig. 6.** Non-parametric test of CRRA and CARA utility.

**Table 3**
Parameter estimates of structural models.

| parameter | DEU | DRDU | QHRDU | HRDU |
|---|---|---|---|---|
| $\rho$ (utility curvature) | 0.273 | 0.512 | 0.517 | 0.514 |
| | (0.268, 0.279) | (0.499, 0.526) | (0.503, 0.531) | (0.5, 0.528) |
| $r$ (discount rate) | 0.059 | 0.141 | 0.111 | 0.239 |
| | (0.056, 0.061) | (0.135, 0.148) | (0.103, 0.119) | (0.211, 0.268) |
| $\gamma$ (prob. sensitivity) | | 0.675 | 0.672 | 0.674 |
| | | (0.655, 0.694) | (0.652, 0.691) | (0.654, 0.693) |
| $\eta$ (prob. pessimism) | | 1.405 | 1.42 | 1.411 |
| | | (1.364, 1.447) | (1.378, 1.462) | (1.369, 1.452) |
| $\beta$ (<1: present bias) | | | 0.972 | |
| | | | (0.967, 0.977) | |
| $\zeta$ (hyperbolicity) | | | | 1.788 |
| | | | | (1.201, 2.376) |
| $\sigma$ (noise) | 0.002 | 0.008 | 0.008 | 0.008 |
| | (0.002, 0.002) | (0.007, 0.009) | (0.007, 0.009) | (0.007, 0.009) |
| max LL | -37348.93 | -36130.75 | -36073.05 | -36066.97 |

95% confidence intervals in parentheses below the estimates.

ratio test). The sensitivity parameter $\gamma$ is clearly smaller than 1, indicating an inverse-S shaped probability weighting function. This shows that the estimation of functions of this type is robust to using a method in which systematic noise would tend to reverse inverse-S weighting. We also find a considerable degree of probabilistic pessimism, captured by $\eta > 1$.

Fig. 7 depicts the probability weighting function estimated in the DRDU model (functions estimated in the two subsequent models with probability weighting are very similar). The function clearly exhibits an inverse-S shape, confirming previous results (Tversky and Kahneman, 1992; Wu and Gonzalez, 1996; Abdellaoui, 2000). At the same time, the inflection point falls relatively low, and the degree of probabilistic pessimism is relatively high. This may be due to one of two possible factors. One, we used real incentives of up to €500, which are higher than in most experiments. Given that probability weighting may not be completely independent of stake sizes (Fehr-Duda et al., 2010; Bouchouicha and Vieider, 2017), this may result in a lower probability weighting function. Two, the particular type of MPL used may produce systematically higher estimates of risk aversion than other measuring techniques. Given the MPLs setup, there is less space in most lists to detect risk seeking than risk aversion, and this is especially true for the lists with small EV switching probabilities. While this may bias our estimates against inverse-S, it is the price to be paid to show that these patterns are strong enough to overpower any possible confound derived from random switching. The corollary of the high level of pessimism we find is a utility function that exhibits less curvature in the DRDU model than the one estimated under DEU. This, in turn, also impacts the estimate of the discount rate, which at over 14% is now more than twice as high as the one estimated under DEU.

The model in column 3 relaxes the assumption of constant discounting, and instead allows discounting to be quasi-hyperbolic. We denote it by QHRDU, for Quasi Hyperbolic Rank Dependent Utility. This further improves model fit ($\chi^2(1) = 135.9$, $p < 0.000$; likelihood ratio test). The $\beta$ parameter is smaller than 1, indicating present bias. Finally, column 4 presents an RDU model combined with a fully flexible hyperbolic discount function, called HRDU for Hyperbolic RDU. The HRDU model has a higher likelihood than QRDU, but the difference is not significant. ($z = -0.341$, $p = 0.367$; Vuong test).

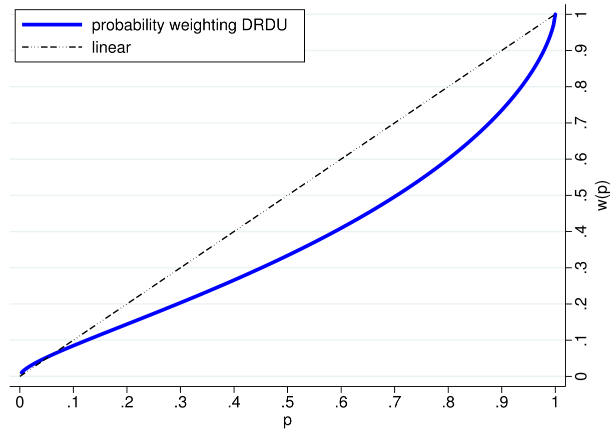



**Fig. 7.** Probability weighting function estimated in DRDU model.

**Table 4**
Individual-level estimates of the QHRDU, HRDU and CSRDU models.

| Parameter | 1stQ | Median | 3rdQ | Mean | Nr. significant |
|---|---|---|---|---|---|
| QHRDU model | | | | | |
| $\rho$ (utility curvature) | 0.27 | 0.44 | 0.46 | 0.44 | 87 |
| $\gamma$ (prob. sensitivity) | 0.52 | 0.79 | 0.90 | 0.79 | 64 |
| $\eta$ (prob. pessimism) | 1.00 | 1.37 | 1.96 | 1.37 | 70 |
| $r$ (discount rate) | 0.02 | 0.08 | 0.19 | 0.08 | 67 |
| $\beta$ (<1: present bias) | 0.97 | 0.99 | 0.98 | 0.99 | 42 |
| HRDU model | | | | | |
| $\rho$ (utility curvature) | 0.25 | 0.43 | 0.46 | 0.43 | 89 |
| $r$ (discount rate) | 0.04 | 0.15 | 0.84 | 0.15 | 72 |
| $\gamma$ (prob. sensitivity) | 0.52 | 0.79 | 0.90 | 0.79 | 65 |
| $\eta$ (prob. pessimism) | 1.00 | 1.36 | 1.96 | 1.36 | 72 |
| $\zeta$ (hyperbolicity) | 0.00 | 0.45 | 15.67 | 0.45 | 18 |
| CSRDU model | | | | | |
| $\rho$ (utility curvature) | 0.26 | 0.47 | 0.47 | 0.47 | 80 |
| $\gamma$ (prob. sensitivity) | 0.58 | 0.74 | 0.89 | 0.74 | 65 |
| $\eta$ (prob. pessimism) | 1.00 | 1.31 | 1.88 | 1.31 | 71 |
| $a$ (impatience) | 0.00 | 0.10 | 0.21 | 0.10 | 48 |
| $b$ (time-sensitivity) | 0.62 | 0.86 | 1.41 | 0.86 | 43 |

### 3.3. Individual estimates

The results discussed up to this point were derived from aggregate estimates of the choice data. However, as it is well known, there is considerable heterogeneity in individual preferences. Table 4 presents summary statistics of these estimates for QHRDU and HRDU, the two models that provide the best fit at the aggregate level. We also include a module using the Ebert and Prelec (2007) constant sensitivity function to capture discounting (we correspondingly label the results CSRDU). In addition to the decreasing impatience captured by the hyperbolic models, this model allows to account for increasing impatience, a preference profile that has been observed in other studies (Abdellaoui et al., 2010; Attema et al., 2010). The models converged for all subjects when flexible start values were used for the maximum-likelihood search. In addition to descriptive statistics of the distribution of estimates, the Table reports the number of statistically significant parameter estimates. The significance for each parameter is measured against a DEU benchmark with no discounting, i.e. against 1 for utility curvature, probabilistic sensitivity, probabilistic pessimism, the present-bias parameter in the QHRDU model, and the time-sensitivity parameter in the CSRDU model; and against 0 for the discount rate, noise, and the hyperbolicity parameter.

There are some interesting features to note. Approximately 65% of the subjects exhibit probabilistic insensitivity across specifications. At the same time, close to 70% of the subjects exhibit a pessimism parameter different from 1. Overall, for 15% of the subjects, both sensitivity and pessimism were not significantly different from 1. This provides us with a rough estimate of the number of subjects for whom we cannot reject the discounted expected utility decision model. The number of subjects following expected utility in our setup is in fact similar to the proportion of EU followers estimated by Bruhin et al. (2010) in their finite mixture model. In terms of time preferences, we find that we can reject the null of non-hyperbolic preferences for approximately 40% of the subjects according to the quasi-hyperbolic model and 20% of the subjects under the fully hyperbolic model. According to a Vuong test evaluated at the 5% significance level, the quasi-hyperbolic model is a better fit than the hyperbolic model for 14 of the subjects. The fully hyperbolic model is a better fit for 9 subjects and the fit of the two models cannot be statistically distinguished for 77 subjects. Interestingly, we also estimate a time-sensitivity parameter that is significantly greater than 1 for 11 subjects. This is an indication of increasing impatience, with a proportion that is in line with findings in some of the studies cited above.

## 4. Discussion

It has been assumed for a long time that linear utility and the absence of risk are necessary for obtaining tractable estimates of time discounting. The supposed absence of risk could lead to distortions in the estimated functions, given that the future is inherently risky (Keren and Roelofsma, 1995; Weber and Chapman, 2005; Halevy, 2008; Gerber and Rohde, 2010; Epper et al., 2011). In turn, ignoring utility curvature could lead to the over-estimation of discounting if the curvature is truly significant. A procedure first proposed by Chapman (1996) was borrowed by Andersen et al. (2008) to correct discounting using utility curvature estimated under risk assuming expected utility. Our results clearly show that this procedure could lead to an over-correction, which would artificially lower estimated discount rates and thus do more harm than good.

The correction we used is based on a model that applies probability weighting to our risky data, thus providing a far better fit. However, this does not imply that our model is de facto “the right one”. For instance, some recent evidence indicates that utility under risk and utility over time may systematically differ, both when risk preferences are estimated under expected utility (Andreoni and Sprenger, 2012), and when they are estimated with probability weighting (Abdellaoui et al., 2013) as we do in the present paper. If inter-temporal utility were to exhibit less concavity than utility under risk—as in the works cited above—then even our correction allowing for probability weighting may still be excessive, thus resulting in a lower bound for estimated discount rates. The present experiment was not set up to test this issue directly, which, in itself, deserves further study.

Finally, we showed that inverse-S shaped probability weighting is stable when applied to the type of MPLs we used. This is important, inasmuch as systematic noise in the form of random switching (or switching towards the middle of a list) could potentially distort the estimates of probability weighting. In this specific design, however, the bias would work *against* inverse-S shaped probability weighting. The fact that we replicated the typical inverse-S shape thus shows the stability of this empirical pattern. Some studies have reported different shapes of probability weighting, including the reverse pattern of S-shaped probability weighting. For instance, Harrison et al. (2010) reported S-shaped probability weighting from four developing countries. Andersen et al. (2014) and Andersen et al. (2017) reported S-shaped probability weighting estimated based on the same type of choice lists used in this paper. The findings in these two papers are most likely due to a poor discriminatory power between utility curvature and probability weighting, given the narrow range of expected value switching probabilities in the stimuli, and the potential presence of noise. The findings in the first study are probably driven by the restrictive assumption of a 1-parameter probability weighting function.[5] Together, these studies underline the importance of explicitly designing experimental stimuli in a way that allows the different dimensions to be identified. Estimating complex models on data that are not especially designed for that purpose is bound to generate biased inferences if the resulting estimations are accepted without question. L'Haridon and Vieider (2018) showed that probabilistic sensitivity is one of the few universal behavioral patterns in student populations from 30 countries. Vieider et al. (2018) generalized this finding to a representative sample of a rural population from Ethiopia. We thus conclude that—notwithstanding some claims to the contrary—inverse-S shaped probability weighting is alive and in good shape.

## 5. Conclusion

This paper presented results from a comprehensive multiple price list experiment to elicit risk and time preferences in an integrated framework. Using a variation on a popular multiple price list design, we estimated time preferences together with risk preferences. Introducing risk may be more realistic than the artificial certainty assumed by the majority of elicitation designs of inter-temporal choice to date. In addition, we designed the choice lists with the explicit goal to allow us to separate utility curvature from probability weighting. To do so, we introduced orthogonality between stakes and probabilities, and scanned the entire probability interval.

For time preferences, we found clear evidence for present bias and hyperbolic behavior. This evidence is contrary to some preceding studies that found that present bias disappears once risk is added (Keren and Roelofsma, 1995; Weber and Chapman, 2005). We found the estimated yearly discount rate to be as low as 6% when adopting discounted expected utility. Adopting a rank-dependent formulation correcting for probability weighting, however, more than doubled the estimated discount rate to 14%. As far as risk preferences are concerned, we found a clear inverse-S shaped pattern, notwithstanding the bias against inverse-S that we built into our lists.

[5] The shape of the function estimated by Harrison et al. (2010) depends crucially on the functional form assumption, with different functional assumptions resulting either in an S-shape, and inverse S-shape, or consistent pessimism. The S-shape emerges only under a 1-parameter form proposed by Tversky and Kahneman (1992), which supports the point we are making.

## Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.geb.2019.03.001.

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